Groups, rings and the Yang-Baxter equation 2023

19.06.2023 - 23.06.2022 — Blankenberge (Belgium)

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Groups, rings and the Yang-Baxter equation 2023

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Foreword

The conference about Groups, Rings and the Yang-Baxter equation 2023, held at Corsendonk Duinse Polders in the beautiful town of Blankenberge, Belgium, is a sequel to the meetings held in 2017 and 2019 at Corsendonk Sol Cress, Spa, Belgium focusing on recent developments in and the interplay between the areas of ring theory and group theory, with a focus on the methods involved in their study and applications to other areas, mainly related to the celebrated Yang-Baxter equation.

Ilaria Colazzo
Arne Van Antwerpen
Leandro Vendramin

Index of Talks

Monday, June 19

Tuesday, June 20

Wednesday, June 21

Thursday, June 22

Friday, June 23

List of Speakers

Zesting link invariants

09:00
09:50

Julia Plavnik

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Modular categories arise naturally in many areas of mathematics, such as conformal field theory, representations of braid groups, quantum groups and Hopf algebras, and low dimensional topology, and they have important applications in condensed matter physics. These categories give rise to representations of the braid group and other motion groups and also knot and link invariants.

It was conjectured that modular categories were determined by its modular data (S- and T-matrices). In 2017, Mignard and Schauenburg presented a family of counterexamples to this conjecture, which led to the study of link invariants beyond modular data to distinguish these categories. In this talk, I will introduce the definition and some examples of a modular category as well as the zesting construction and some of its interesting properties. I will also discuss how the zesting construction is related to the family of Mignard-Schauenburg counterexamples. To better understand this relation, we look into how zesting affects link invariants such as the W-matrix and the B-tensor.

Classification of binary bi-braces and their group of automorphisms

10:00
10:30

Roberto Civino

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Starting from an n-dimensional vector space $(V,+)$ over $\mathbb{F}_2$, we define a class of group operations $\circ$ whose corresponding group of translations is elementary abelian and regular. With the idea of using the new operation to detect undesired biases in the distribution of differences in a secure block cipher, we select $\circ$ in such a way the resulting structure $(V,+,\circ)$ turns out to be a brace with a mutual normalization property of the two translation groups. We call such structures bi-braces and we show how understanding the group of automorphisms of structures as above is crucial in the application to cryptanalysis. We introduce an equivalence relation which allows the classification of bi-braces in terms of binary skew symmetric matrices and their isometry groups.

Growth, dynamics and geometry of noncommutative algebras

11:00
11:30

Be'eri Greenfeld

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We discuss the interaction between symbolic dynamics and noncommutative rings and their geometry, with an emphasis on growth restrictions, amenability and proalgebraic varieties. As a benefit of this interaction, we show how one solves some open problems in both worlds.

This talk is partially based on joint works with J. Bell and with E. Zelmanov.

11:40
12:00

Indecomposable solutions of the Yang–Baxter equation and generators of skew braces

Senne Trappeniers
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Left braces, and later skew left braces, were introduced as a tool to study and produce solutions of the set-theoretic Yang–Baxter equation. Ever since then, this connection remains one of the driving forces behind research on skew braces.

In this talk, based on joint work with M. Castelli, we focus on how the number of orbits, and hence the indecomposability, of a solution of the YBE is related to subsets of its permutation skew brace which generate it as a strong left ideal. We will then introduce related notions of generating sets of skew braces and discuss how, under certain conditions, these different notions coincide and tell us something about subsolutions of associated solutions. If time permits, we will discuss applications of the above to certain classes of solutions.

12:10
12:30

Irreducible representations of the Hecke–Kiselman algebras

Magdalena Wiertel
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To every finite oriented graph θ with n vertices one can associate a finitely presented monoid HK_θ, called the Hecke–Kiselman monoid. It is a monoid generated by n idempotents with relations of the form $xy = yx$ or $xyx = yxy = xy$, depending on the edges between vertices x and y in θ. By the Hecke–Kiselman algebra we mean the monoid algebra $K[HK_\theta]$ of the monoid HK_θ over an algebraically closed field K.

We focus on the representations of the Hecke–Kiselman algebras. The radical is described in case the algebra $K[HK_\theta]$ satisfies a polynomial identity. The latter condition can be expressed in terms of the graph θ. All irreducible representations, and the corresponding maximal ideals, are then characterized for this case. Every representation either is one-dimensional and it comes from an idempotent in the Hecke–Kiselman monoid or it comes from certain semigroups of matrix type arising from HK_θ. The result shows a surprising similarity to the classical theorems on the representations of finite semigroups.

14:00
14:50

Zesting of Verlinde Modular Data

César Galindo
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A Modular Tensor Category, while complex, has associated a highly constrained combinatorial invariant that nearly serves as a complete invariant in practice.

Among modular categories, the Verlinde Modular Categories—denoted as $\mathcal{C}(\mathfrak{g}, q)$ and associated with a simple Lie algebra $\mathfrak{g}$ and a root of unity q—are perhaps the most significant due to their diverse applications in both low-dimensional topology and quantum computing, among other areas. These modular categories are constructed as the semisimplification of the tilting modules of the quantum group $U_q(\mathfrak{g})$.

The zesting construction provides a way for generating new modular categories from an existing one. In this presentation, recent advancements in applying the zesting construction to Verlinde modular categories are discussed. We offer a comprehensive description of the zestings and the modular data related to the Zesting of Verlinde modular categories. This work is a collaboration with Giovanny Mora.

On solutions of the YBE subjected to a choice of elements

15:00
15:30

Bernard Rybołowicz
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In 2007 W. Rump discovered an interesting connection between algebraic structures, which he named braces, and the set-theoretic Yang-Baxter equation. In the talk, I will start with a solution and a group. I will put some constraints on the operation. That will allow me to construct an algebraic structure called a near brace. After that, I will discuss how one can find solutions by looking at near braces/skew braces from the affine point of view.

Centrally nilpotent skew braces

15:40
16:10

Arne Van Antwerpen
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This talk will be based on joint work with Eric Jespers and Leandro Vendramin. The class of multipermutation solutions is a particularly interesting class of solutions of the celebrated Yang-Baxter equation (coming from mathematical physics) with a beautiful combinatorial structure. It was shown that this class corresponds to right nilpotent skew left braces of nilpotent type. In this talk we delve deeper into this class of skew left braces and identify the class of centrally nilpotent skew braces. We discuss that these behave very similar to nilpotent groups and will identify several possible central series for these objects. If time permits, we use this class to illustrate several other key concepts of skew left braces. The talk will be rife with examples and exciting open problems.

Solutions to the YBE: cabling and decomposability

16:40
17:10

Ilaria Colazzo
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This talk is based on joint work with Arne Van Antwerpen. We will focus on bijective non-degenerate solutions to the Yang-Baxter equation (YBE). We will introduce the cabling for bijective non-degenerate solutions and show that this is a useful tool for dealing with indecomposability.

The Isomorphism Problem for Rational Group Algebras of Metacyclic Groups

17:15
17:35

Àngel García Blàzquez
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The Isomorphism Problem for group rings with coefficients in a ring R asks whether the isomorphism type of a group G is determined by its group ring RG.

We discuss the Isomorphism Problem in the case where $R = \mathbb{Q}$ and the groups are metacyclic. We explain the proof of the positive answer to the Isomorphism Problem in this case.

17:40
18:00

Hopf-Galois Structures and Transitive Subgroups

Andrew Darlington
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In this talk, I will give a brief overview of objects known as Hopf-Galois structures, which in some sense describe a Galois theory for separable (and not necessarily normal) field extensions. It turns out that the task of finding and describing Hopf-Galois structures may be done entirely group-theoretically, by looking at transitive subgroups of the holomorph of a group of the same order as the degree of the extension. I will touch on some partial answers to a few open problems which arise within this theory, as well as known connections with other algebraic objects such as skew braces in the case that the extension is Galois.

Modular Isomorphism Problem - progress, solution and open challenges

09:00
09:50

Leo Margolis
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Say we are given only the R-algebra structure of a group ring RG of a finite group G over a commutative ring R. Can we then find the isomorphism type of G as a group? This so-called Isomorphism Problem has obvious negative answers, considering e.g. abelian groups over the complex numbers, but more specific formulations have led to many deep results and beautiful mathematics. The last classical open formulation was the so-called Modular Isomorphism Problem: Does the isomorphism type of kG as a ring determine the isomorphism type of G as a group, if G is a p-group and k a field of characteristic p?

Starting with an overview on the state of knowledge on general Isomorphism Problems and the modular one in particular, I will present a negative solution found rather recently, but also present positive structural results and several problems remaining open.

Counterexamples to the Zassenhaus conjecture on simple modular Lie algebras

10:00
10:30

Pilar Páez-Guillán
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Historically, the study of the (outer) automorphism group of a given group (free, simple...) has interested group-theorists, topologists and geometers, and consequently it is also of great importance in the Lie algebra theory. In this talk, we will compare the Schreier and Zassenhaus conjectures on the solvability of $\mathrm{Out}(G)$ (resp. $\mathrm{Out}(L)$, the group of outer automorphisms (resp. the Lie algebra of outer derivations) of a finite simple group G (resp. a finite-dimensional simple Lie algebra L). While the former is known to be true as a consequence of the classification of finite simple groups, the latter is false over fields of small characteristic $p = 2, 3$. We will finish the talk by presenting a new family of counterexamples to the Zassenhaus conjecture over fields of characteristic $p = 3$, as well as commenting some advances for $p = 2$.

The representation theory bridge from the Bogomolov multiplier to class preserves

11:00
11:30

Geoffrey Janssens
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To any finite group G we will associate two invariants that have a 'rigidity' flavour. A first will be $Out_c(G)$, the group of class-preserving outer automorphisms of G. The second, given a faithful representation V of G, is a birational invariant of the quotient variety V/G called the Bogomolov multiplier $B_0(G)$. In the first part of the talk we will recall the definition of both and why they express a type of rigidity. The rest of the talk will be centred around Kang-Kunyavskii's question concerning the link between these two invariants. More precisely, we will sketch the bridging role of the tensor autoequivalences of $Rep_{\mathbb{C}}(G)$, the complex representations of G, and the cohomology ring $H^*(G, R)$ with $R = \mathbb{Z}$ or $\mathbb{F}_p$. This talk is based on ongoing work with Urban Jezernik.

11:40
12:00

A common divisor graph for skew braces

Silvia Properzi
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In the combinatorial study of solutions to the Yang–Baxter equation (YBE), recently introduced ring-theoretical objects play a fundamental role: skew braces. A skew brace is a set with two group operations + and $\circ$ satisfying a compatibility condition. In a skew brace $(A,+,\circ)$, the group $(A,\circ)$ acts on $(A,+)$ by automorphism via the λ-map, $\lambda\colon (A,\circ) \to \mathrm{Aut}(A,+)$. This action is involved in the (universal) construction of the set-theoretic solutions to the YBE provided by skew braces. Furthermore, the λ-action deeply influences the structure of a finite skew brace, as e.g. ideals are λ-invariant normal subgroups (for both group structures). Motivated by similar ideas in representation theory of finite groups and by the work of of Bertram, Herzog, and Mann, we study a common divisor graph: the simple undirected graph whose vertices are the non-trivial λ-orbits and two vertices are adjacent if their sizes are not coprime. We provide some examples and prove that it has at most two connected components and that, in the connected case, its diameter is at most four. The main result is a complete classification of finite skew braces with a one-vertex graph. In particular, the number of non-isomorphic skew braces of size $2^m d$ (with d odd) whose graph has only one vertex is $m \cdot a(d)$ if m 3 and $2a(d)$ if m 4, where $a(d)$ is the number of isomorphism classes of abelian groups of order d.

Joint work with Arne Van Antwerpen.

12:10
12:30

The modular isomorphism problem and abelian direct factors

Diego García Lucas
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Let p be a prime, G be a finite p-group, k the field with p elements, and kG the group algebra of G over k. The modular isomorphism problem (MIP) asks whether the isomorphism type of G can be recovered from the structure of kG as k-algebra. More generally, and since this question is now known to have a negative answer (at least for $p = 2$), the problem can be reformulated as: what information about the group G can be recovered from kG? We show that both the isomorphism type of a maximal abelian direct factor A of G and the isomorphism type of kN (as k-algebra), where N is such that $G = A \times N$, can be recovered from kG. This shows that MIP can be reduced to the same problem over groups without abelian direct factors.

14:00
14:50

Better bounds on growth and diameter in classical Chevalley groups

Harald Helfgott
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Joint with Jitendra Bajpai and Daniele Dona.

Babai's conjecture states that, for any finite simple non-abelian group G, the diameter of G is bounded by $(\log|G|)^C$, where C is a constant. A series of results since (Helfgott, 2005-2008) has given us cases of Babai's conjecture for different families of groups. However, for linear algebraic groups G, the dependence of C on the rank of G has been very poor (exponential-tower).

(a) How much can one improve the bound, while keeping the general inductive idea in Larsen-Pink (1998-2011) (which they used to classify subgroups in SL_n; generalized for use in this context by Breuillard-Green-Tao, 2010-2011) or in (Pyber-Szabo, 2010-2016)?

(b) Can one change the strategy and prove a yet better bound?

On (a), we will show the main ideas that have allowed us to make C exponential on the rank, while keeping an inductive argument that resembles those used before. (Actually keeping the Larsen-Pink inductive process gives a worse but still exponential bound, if combined with our other improvements.)

We will also discuss (b): changing the strategy - from general dimensional bounds as in Larsen-Pink - we have been able to make C polynomial on the rank.

Strong semilattices of skew braces

15:00
15:30

Marzia Mazzotta
joint work with Francesco Catino and Paola Stefanelli

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The challenging problem posed by Drinfel'd in 1992 of determining all set-theoretic solutions of the Yang-Baxter equation is still open. In this direction, one of the major research areas of the last twenty years concerns the study of brace-like structures for the solutions that can be associated and for their similarities with rings.

In this talk, we present the algebraic structure of dual weak brace that is a triple $(S,+,\circ)$ having $(S,+)$ and $(S,\circ)$ as Clifford semigroups and satisfying the relations

$$a\circ(b+c)=a\circ b-a+a\circ c \qquad \& \qquad a\circ a^{-}=-a+a,$$

for all $a,b,c\in S$, where $-a$ and a^{-} denote the inverses of a with respect to $+$ and $\circ$, respectively. In particular, if $(S,+)$ and $(S,\circ)$ are groups, then $(S,+,\circ)$ is a skew brace. Every dual weak brace gives S rise to a solution r_S close to being bijective and non-degenerate which we prove is a strong semilattice of bijective and non-degenerate solutions r_α coming from specific skew braces B_α. Such skew braces are those realizing S, since we show that it is a strong semilattice $[Y,B_\alpha,\phi_{\alpha,\beta}]$ of skew braces B_α, with $\alpha\in Y$.

This talk is based on a work in progress with F. Catino and P. Stefanelli.

Advances on Quillen's conjecture

15:40
16:10

Kevin Piterman

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The study of the p-subgroup complexes began motivated by group cohomology and equivariant cohomology of topological spaces "modulo the prime p". For example, Kenneth Brown proved that the reduced Euler characteristic of this complex is divisible by the size of a Sylow p-subgroup, giving rise to a sort of "Homological Sylow theorem". Later, he showed that the mod-p equivariant cohomology of the p-subgroup complex of a finite group coincides with the mod-p cohomology of the group. Deeper relations with finite group theory, representation theory, and finite geometries were also explored. For instance, uniqueness of certain simple groups, finite geometries for sporadic groups, Lefschetz modules, and, more recently, endotrivial modules.

In 1978, Daniel Quillen conjectured that the poset of non-trivial p-subgroups of a finite group G is contractible if and only if G has non-trivial p-core. Quillen established the conjecture for solvable groups and some families of groups of Lie type. The major step towards the resolution of the conjecture was done by Michael Aschbacher and Stephen D. Smith at the beginning of the nineties. They roughly proved that if $p>5$ and G is a group of minimal order failing the conjecture, then G contains a simple component $\mathrm{PSU}(n,q^2)$ failing a certain homological condition denoted by $(\mathcal{QD})$ (namely, the top-degree homology group of its p-subgroup poset does not vanish).

In this talk, I will present recent advances in the conjecture, with a particular focus on the prime $p = 2$, which was not covered by the methods developed by Aschbacher-Smith. In particular, we show that the study of the conjecture for the prime $p = 2$ basically reduces to studying $(\mathcal{QD})$ on the poset of p-subgroups of certain families of classical groups. Part of this work is in collaboration with S.D. Smith.

16:40
17:10

Derived-indecomposable solutions, Skew braces and Presentations

Marco Trombetti
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Derived-indecomposable solutions are controlled by skew braces in which every element has finitely many conjugates. Thus, if B is a skew brace, the size (with respect to B) of the set $F(B)$ of all elements of B having finitely many conjugates could be used as a measure of how good is this controlling. It is unknown if the set of all elements of a skew brace with finitely many conjugates is actually a sub-skew brace (or even an ideal), but it turns out for instance that the structure skew brace does always contain an ideal I which is contained in $F(B)$ and is such that B/I is finite. The aim of the talk is to introduce the audience to a method proving this result, and having some other consequences for the general theory of (possibly infinite) skew braces.

17:15
17:35

On the Donald-Flanigan conjecture, related to deformations of group algebras

Ariel Amsalem
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The J.D. Donald and F. J. Flanigan conjecture asserts that every group algebra has a separable deformation. In this talk, I will discuss about what has been done so far on this topic, and I will present a new infinite family of group algebras that fulfill the conjecture.

17:40
18:00

On the number of quaternionic and dihedral braces

Fabio Ferri
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Let $m > 2$ be an integer and let $q(4m)$ denote the number of isomorphism classes of braces of order $4m$ with quaternionic multiplicative group. Guarnieri and Vendramin conjectured that $q(4m)$ only depends on the exact power of 2 dividing m, and claimed the exact formula of its value following computer calculations. Rump recently proved the formula if m is a power of 2. Through an explicit description of regular quaternionic subgroups of the holomorph of an abelian group, we prove the conjecture in its full generality. We also provide an expression for the analogue number $d(4m)$ of isomorphism classes of braces with dihedral multiplicative group. This is joint work with Nigel Byott.

Graphs on groups, rings and maybe YBE solutions

09:00
09:50

Peter Cameron
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The connection between graphs and groups goes back to Cayley in the 19th century. More relevant to my topic is the seminal paper of Brauer and Fowler from 1955, in which they used the commuting graph of a finite simple group to show that there were only finitely many such groups which have a prescribed involution centraliser. This was perhaps the first step on the thousand-mile journey to the Classification of Finite Simple Groups.

Since then, many other graphs have been defined on groups (including the power graph and generating graph), and similar definitions have been made for rings (including the zero-divisor graph). I have been involved in this with a large group of mathematicians, mostly in India, as a result of an on-line research discussion group run during the pandemic.

The main features of this particular interaction between graphs and algebraic structures are:

- Like Brauer and Fowler, we can use graphs to understand groups better.
- Some of the graphs that arise are interesting for graph theory and its applications.
- Interesting classes of groups can be defined by graphs, either by requiring that two particular graphs on a group (such as the power graph and commuting graph) are equal, or by asking when the graph belongs to a particular class (such as cographs or perfect graphs).

I hope that similar analysis of YBE solutions may be possible using graphs, though at present I don't have much to report on this.

Stability and ω-Categoricity of Skew Braces

10:00
10:30

Maria Ferrara
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What does a skew brace look like from the first-order logic point of view?

The aim of this talk is to introduce the audience to a recent joint work with Marco Trombetti (Università degli Studi di Napoli Federico II) and Frank O. Wagner (Université de Lyon) in which we tried to understand the first-order theory of skew braces (and related nilpotency concepts) with particular attention to ω-categorical, stable skew braces.

Central nilpotency and solubility of skew braces and solutions of the YBE

11:00
11:30

Vicent Pérez-Calabuig
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The study of non-degenerate set-theoretic solutions of the Yang-Baxter equation calls for a deep understanding of the algebraic structure of a skew left brace. Indeed, nilpotency in skew left braces turns out to be a good framework to understand one of the most significant classes of non-degenerate solutions: the multipermutation one. Bearing in mind that soluble groups are the ones with a rich normal subgroup structure, we introduce a suitable notion of solubility of skew left braces to provide a proper framework for studying decomposability of solutions. Our main goal in this talk is to present a detailed study of central nilpotency and solubility of skew left braces and analyse their impact on solutions of the Yang-Baxter equation. The influence of the solubility of solutions leads naturally to the concept of multidecomposability. The relation between multidecomposable and multipermutation solutions is also given.

11:40
12:30

Indecomposable and simple solutions of the YBE

Jan Okniński

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Finite involutive non-degenerate set-theoretic solutions of the Yang-Baxter equation are discussed. We focus on indecomposable solutions and on simple solutions. Some of the known constructions of simple solutions are presented. Then a recent result on solutions of square-free cardinality is discussed. The new results of the talk come from recent joint papers with Ferran Cedó.

Cactus groups, partial solutions to the YBE, and right-angled Artin groups

09:00
09:50

Victoria Lebed

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One of the multiple ways of interpreting the set-theoretic Yang-Baxter equation is as the key condition for the deformation $\langle X \mid xy = r(x,y) \text{ for } x,y \in X\rangle$ of the free abelian group $\langle X \mid xy = yx \text{ for } x,y \in X\rangle$ on a set X to inherit its nice properties. Another way to generalise free abelian groups is to impose commutativity on certain pairs of elements of X only. This yields the celebrated right-angled Artin groups (RAAGs), which, in spite of the seemingly elementary definition, still present thrilling open questions. This talk will focus on cactus groups, which mix the two generalisations above: they are defined by twisted commutation relations for certain pairs of generators only. We will describe an injective group 1-cocycle from any cactus group to a certain RAAG (which should look very familiar to brace theorists!), and exploit it for solving the word problem for cactus groups, and for computing their torsion and center. (Joint work with P. Bellingeri and H. Chemin.)

Fuchs' problem for finitely generated groups of units

10:00
10:30

Ilaria Del Corso

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The study of the group of units of a ring is an old problem. The first general result is the classical Dirichlet's Unit Theorem (1846), which describes the group of units of the ring of integers $\mathcal{O}_K$ of a number field K, proving that $\mathcal{O}_K = \mathbb{Z}/2n\mathbb{Z} \times \mathbb{Z}^g$ where n and g are determined from K.

In 1940 G. Higman discovered a perfect analogue of Dirichlet's Unit Theorem for a group ring $\mathbb{Z}[T]$ where T is a finite abelian group: $(\mathbb{Z}[T])^* = \pm T \times \mathbb{Z}^g$ for a suitable explicit constant $g = g(T)$.

In 1960 Fuchs in [Abelian Groups, (Pergamon, Oxford, 1960); Problem 72] posed the following problem.

Characterize the groups which are the groups of all units
in a commutative and associative ring with identity.

In this talk, I will consider Fuchs' problem for finitely generated abelian groups. In the first part, I will present a complete characterisation of groups of units realised in certain fixed classes of rings, namely integral domains, torsion-free rings and reduced rings. This is achieved through a very careful study of the orders of cyclotomic K-algebras, where K is a suitable number field.

In the second part I will present some results obtained in an ongoing project with Lorenzo Stefanello, in which we have made some progress on the Fuchs' question for general rings, using recent results on module braces.

11:00
11:30

The Diamond Lemma through homotopical algebra

Pedro Tamaroff

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The Diamond Lemma is a result indispensable to those studying associative (and other types of) algebras defined by generators and relations. In this talk, I will explain how to obtain a new approach to this celebrated result through the homotopical algebra of associative algebras: we will see how every multigraded resolution of a monomial algebra leads to its own Diamond Lemma, which is hard-coded into the Maurer–Cartan equation of its tangent complex. For the reader familiar with homotopical algebra, we hope to provide a conceptual explanation of a very useful but perhaps technical result that guarantees uniqueness of normal forms through the analysis of overlapping ambiguities. For a reader familiar with Gröbner bases or term rewriting theory, we hope to offer some intuition behind the Diamond Lemma and at the same time a framework to generalize it to other algebraic structures and optimise it. This is joint work with Vladimir Dotsenko.

11:40
12:00

The normal complement problem in group algebras

Himanshu Setia

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Let S_n be the symmetric group and A_n be the alternating group on n symbols. In this talk, we have proved that if F is a finite field of characteristic $p > n$, then there does not exist a normal complement of S_n (n is even) and A_n (n 4) in their corresponding unit groups $\mathcal{U}(FS_n)$ and $\mathcal{U}(FA_n)$. Moreover, if F is a finite field of characteristic 3, then A_4 does not have normal complement in the unit group $\mathcal{U}(FA_4)$.

This is a joint work with Dr. Manju Khan.

12:10
12:30

Skew braces and the Hopf–Galois correspondence

Lorenzo Stefanello

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Hopf–Galois theory, a generalisation of Galois theory in which the action of a Galois group is replaced by a suitable action of a Hopf algebra, has been showed to be valuable in dealing with classical arithmetic problems in a more general context.

An open problem in Hopf–Galois theory regards the surjectivity of the Hopf–Galois correspondence; the well-know bijective Galois correspondence between subgroups of the Galois group and intermediate fields can be generalised to Hopf–Galois structures, but the correspondence one obtains is injective but not necessarily surjective. Very few examples where this correspondence is surjective are known; it is of interest to find new ones, and to understand more in general why it should be (or not be) surjective.

The goal of this talk it to approach this problem using the point of view of skew braces. We show that using a new version of connection between skew braces and Hopf–Galois structures, recently developed with S. Trappeniers, we can translate the problem of the surjectivity of the Hopf–Galois correspondence in a natural problem in skew brace theory, thereby deriving several new examples of Hopf–Galois structures for which the Hopf–Galois correspondence is surjective.

Exploring Interconnections among Algebraic Structures

14:00
14:50

Agata Smoktunowicz
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In this discussion, we aim to explore how some connections that exist between various algebraic structures can be applied to theory of braces.

We will delve into the intricate web of connections among different algebraic structures, such as Lie rings, groups, Jordan algebras, braces, pre-Lie algebras, and braided groups.

Throughout our exploration, we will draw upon the contributions of esteemed researchers who have made significant strides in this field. Notable figures such as Lazard, Magnus, Kostrykin, Zelmanov, Shalev, Rump, and Gateva-Ivanova have made invaluable contributions to uncovering these connections, and their work serves as the foundation for our discussion.

Moreover, we will introduce some fresh insights and discoveries that have emerged from recent research. Specifically, we will focus on the intriguing connections between braces and pre-Lie rings. Additionally, we will address some open questions that currently remain unresolved.

Part of this talk is related to a joint work with Aner Shalev.

Skew bracoids and quotients of skew braces

15:00
15:30

Paul Truman
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It is known that the quotient of a skew brace by an ideal is again a skew brace. We show that the quotient of a skew brace by a strong left ideal still exhibits useful and interesting algebraic properties; in fact, it is an example of a more general object, which we term a skew bracoid. We explore applications of these ideas to skew brace theory, Hopf-Galois theory, and the generation of solutions to the Yang-Baxter equation.

Segre products and Segre morphisms in a class of Yang-Baxter algebras

15:40
16:10

Tatiana Gateva-Ivanova
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The quadratic algebras related to set-theoretic solutions of the Yang-Baxter equation are important for both noncommutative algebra and non-commutative algebraic geometry, as they provide a rich source of examples of interesting associative algebras and non-commutative spaces some of which are Artin-Schelter regular algebras. Our work is motivated by the relevance of those algebras for non-commutative geometry, especially in relation to the theory of quantum groups, and inspired by the interpretation of morphisms between non-commutative algebras as "maps between non-commutative spaces". Let (X, r_X) and (Y, r_Y) be finite nondegenerate involutive set-theoretic solutions of the Yang-Baxter equation, and let $A_X = A(K, X, r_X)$ and $A_Y = A(K, Y, r_Y)$ be their quadratic Yang-Baxter algebras over a field K. We find an explicit presentation of the Segre product $A_X \circ A_Y$ in terms of one-generators and quadratic relations. We introduce analogues of Segre maps in the class of Yang-Baxter algebras and find their images and their kernels. The results agree with their classical analogues in the commutative case.

References

[1] Gateva-Ivanova, Tatiana, "Segre products and Segre morphisms in a class of Yang–Baxter algebras." Letters in Mathematical Physics 113.2 (2023): 34.

16:40
17:10

Quotient gradings and IYB groups

Ofir Schnabel
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A grading of an algebra A by a group Γ is a vector space decomposition $A = \bigoplus_{\gamma\in\Gamma} A_\gamma$ such that $A_{\gamma_1} \cdot A_{\gamma_2} \subseteq A_{\gamma_1\cdot\gamma_2}$ for every $\gamma_1, \gamma_2 \in \Gamma$.

A group H of order n induces a natural unique elementary crossed product grading class of the matrix algebra $A = M_n(\mathbb{C})$. This class admits a representative determined by any tuple of distinct elements in H, $(h_1, h_2, \ldots, h_n)$, where the grading is givenby $A_h = \text{span}\{E_{ij} | h = h_i h_j^{-1}\}$. On the other hand, any simple twisted group algebra $\mathbb{C}^\alpha G$ which corresponds to a group of central type G of order n^2 and a nondegenerate cocycle $\alpha \in Z^2(G, \mathbb{C}^*)$ is also equipped with a natural G grading of A. We prove that such an H-elementary crossed product grading class is a quotient grading class of such twisted group algebra if an only if H is an IYB group, that is a multiplicative group of a brace.

Joint work with Yuval Ginosar.

17:15
17:35

Almost Classical Skew Bracoids

Isabel Martin-Lyons
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A skew bracoid is a generalisation of a skew brace consisting of two groups which are related by an action. These objects correspond to Hopf-Galois structures on separable but not necessarily normal extensions of fields, as well as transitive subgroups of the holomorph. We investigate the situation in which the acting group is as a (particular form of) semi-direct product, which aligns with the condition that a related extension of fields is almost classically Galois.

17:40
18:00

Germs and Sylows for structure group of solutions to the YBE

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In 2015 Dehornoy published an article on cycle sets (equivalent to involutive left non-degenerate solutions) introducing a calculus to re-obtain well-known results (I-structure, Garsideness) along with new constructions (the class of a solution, the germ associated to the structure group). We will first briefly mention some results on the class, such as a conjecture on a bound that is verified in a specific case. The main goal is the construction of the Zappa-Szép product of two solutions, allowing us to reduce the problem of classifying all solutions to only the ones with Dehornoy's class a prime power.

On Semisimple PI invariants of finite-dimensional algebras

09:00
09:50

Eli Aljadeff

Technion - Israel Institute of Technology
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Kemer's "Representability theorem" says that if W is an associative PI algebra over a field of characteristic zero then it is PI equivalent to the Grassmann envelope $E(A_2)$ of a finite dimensional super algebra A_2. In case W is affine and PI, or more generally if W satisfies a Capelli identity, then W is PI equivalent to a finite dimensional algebra A.

It is well known (and easy to show) that the finite dimensional algebra A is not determined by its T-ideal of identities (similarly the finite the dimensional super algebra A_2). The purpose of this lecture is to present the following result: The semisimple part of A (resp. of A_2) is "basically" determined by the ideal of identities. Of course I'll explain what "basically" means here. These results may be extended to the group graded setting. Joint work with Karasik.

About non-degenerate involutive solutions of the YBE

10:00
10:30

Raúl Sastriques Guardiola

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As the title suggests, several results about the Yang-Baxter equation will be presented aiming to the state-of-the-art of non-degenerate involutive solutions.

On ring-theoretical properties of Yang–Baxter algebras

11:00
11:30

Łukasz Kubat

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For a finite bijective non-degenerate solution (X, r) of the Yang–Baxter equation it is known that the algebra $\mathcal{A}_K(X, r) = K\langle X \mid xy = uv \text{ if } r(x, y) = (u, v)\rangle$, over a field K, satisfies a polynomial identity, is two-sided Noetherian and has finite Gelfand–Kirillov dimension. Furthermore, if r is involutive then $\mathcal{A}_K(X, r)$ shares many other properties with polynomial algebras in commuting variables (e.g., it is a Cohen–Macaulay domain of finite global dimension).

The aim of this talk is to explain the intriguing relationship between ring-theoretical and homological properties of algebras $\mathcal{A}_K(X, r)$ and properties of finite non-degenerate solutions (X, r) of the Yang–Baxter equation. The main focus is on when such algebras are Noetherian, (semi)prime and representable.

The talk is based on a joint work with I. Colazzo, E. Jespers and A. Van Antwerpen.

11:40
12:00

The Quiver-Theoretic Dynamical YBE

Davide Ferri

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The set-theoretic YBE is understood as a braid relation on a set. This is naturally generalised to the dynamical DYBE: a solution to the DYBE will be a braided quiver.

We shall give an overview on the theory of braided quivers and why it is relevant, and we shall survey some results on the DYBE that generalise known results on the YBE. In particular, we introduce the theory of dynamical braces, highlighting the differences with the non-dynamical theory of braces.

Finally, we sketch the interplay between DYBE and Garside theory, discussing the dynamical analogue of a famous result by F. Chouraqui.

12:10
12:30

Fundamental Superalgebras in PI Theory

Elena Pascucci
joint work with Antonio Giambruno and Ernesto Spinelli

Sapienza Università di Roma

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Fundamental superalgebras are special finite-dimensional superalgebras over an algebraically closed field of characteristic zero defined in terms of certain multialternating graded polynomials. They play a key role in Kemer's Representability Theorem. In the present talk we provide new examples of fundamental superalgebras. Finally we shall give a characterization of fundamental superalgebras in terms of the representation theory of the hyperoctahedral group.

This is based on a joint work with Antonio Giambruno and Ernesto Spinelli.

14:00
14:50

Duality structures on tensor categories coming from vertex operator algebras

Simon Wood

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Modular tensor categories have attracted much attention in the past due to their rich structure. An important source of examples is the representation theory of certain vertex operator algebras (called rational vertex operator algebras). A particularly important property of a modular tensor category is that it is rigid, that is, every object has a dual with corresponding evaluation and coevaluation morphisms. However, representations of a general vertex operator algebra need not be rigid. Instead they admit a more general notion of duality called Grothendieck-Verdier duality. In this talk I will give an overview why Grothendieck-Verdier duality is the right notion of duality to consider for vertex operator algebras and of some of the features of tensor categories admitting Grothendieck-Verdier duality.

Beams and Scaffolds - the art of building modular Garside groups

15:00
15:30

Carsten Dietzel
Vrije Universiteit Brussel
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By results of Chouraqui and Rump, the structure groups of involutive non-degenerate set-theoretic solutions to the Yang-Baxter equation coincide are exactly the Garside groups with a distributive lattice structure. It can be shown that all these groups G decompose canonically as a lattice-theoretic product $G \cong \prod_{i=1}^{k} \mathbb{Z}$.

The situation in case of a Garside group G with a modular lattice structure turns out to be quite similar - each such group contains a canonical distributive subgroup $\mathcal{D}(G)$ G - the distributive scaffold - whose lattice-decomposition $\mathcal{D}(G) \cong \prod_{i=1}^{k} \mathbb{Z}$ is induced by a lattice decomposition $G \cong \prod_{i=1}^{k} \beth_i$ into primary lattices $\beth_i$ which are called the beams of G. In this sense, each modular Garside group contains the structure group of an involutive solution whose lattice-structure also controls the decomposition into beams.

In this talk, I give an outline of the architecture of modular Garside groups, starting with the decomposition of a modular Garside group into beams and ending with a characterization of the beams of dimension 4.

A family of solutions of the Yang-Baxter equation associated to a skew brace

15:40
16:10

Paola Stefanelli
joint work with Marzia Mazzotta and Bernard Rybołowicz
Università del Salento
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It is well-known that skew braces are fundamental algebraic structures for studying set-theoretical solutions of the Yang-Baxter equation. In particular, any skew brace $(B,+,\circ)$ determines a bijective non-degenerate solution r_B that is involutive if and only if B is a brace. Recently, Doikou and Rybołowicz have shown that a new family of solutions can be obtained from any skew brace B by "deforming" the classical one r_B by certain parameters $z \in B$.

This talk aims to show the parameters giving rise to deformed solutions are exactly those belonging to the set

$$\mathcal{D}_r(B) = \{z \in B \mid \forall\, a, b \in B \quad (a+b)\circ z = a \circ z - z + b \circ z\}$$

which we call the right distributor of B and is a subgroup of $(B,\circ)$. Some natural issues concerning such a set will be discussed. These results are based on a joint work with Marzia Mazzotta and Bernard Rybołowicz.

Classical dynamical Yang-Baxter equation in 3d gravity

16:40
17:00

Juan Carlos Morales Parra
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In the talk we will present different methods to obtain classical dynamical r-matrices associated to the Lie algebras of the local isometry groups regarding 3d gravity (including direct solution of the CDYBE, Inonu-Wigner contraction and explicit computation of Alekseev-Meinrenken type). Also, we will show how these classical dynamical r-matrices could be used to define Poisson and quasi-Poisson structures related to phase spaces appearing in the Chern-Simons formulation of 3d gravity.

List of Speakers

V

W

Groups, rings and the Yang-Baxter equation

Schedule 19.06.2023 - 23.06.2022 – Blankenberge (Belgium)

	Monday	Tuesday	Wednesday	Thursday	Friday
09:00 - 09:50	**Plavnik**	**Margolis**	**Cameron**	**Lebed**	**Aljadeff**
10:00 - 10:30	**Civino**	**Páez-Guillán**	**Ferrara**	**Del Corso**	**Sastriques**
10:30 - 11:00	Coffee break	Coffee break	Coffee break	Coffee break	Coffee break
11:00 - 11:30	**Greenfeld**	**Janssens**	**Pérez-Calabuig**	**Tamaroff**	**Kubat**
11:40 - 12:00	**Trappeniers**	**Properzi**	**Okniński**	**Setia**	**Ferri D.**
12:10 - 12:30	**Wiertel**	**Garcia Lucas**		**Stefanello**	**Pascucci**
12:30 - 14:00	Lunch	Lunch	Lunch	Lunch	Lunch
14:00 - 14:50	**Galindo**	**Helfgott**		**Smoktunowicz**	**Wood**
15:00 - 15:30	**Rybołowicz**	**Mazzotta**		**Truman**	**Dietzel**
15:40 - 16:10	**Van Antwerpen**	**Piterman**		**Gateva-Ivanova**	**Stefanelli**
16:10 - 16:40	Coffee break	Coffee break		Coffee break	Coffee break
16:40 - 17:10	**Colazzo**	**Trombetti**		**Schnabel**	
17:15 - 17:35	**García Blàzquez**	**Amsalem**		**Martin-Lyons**	
17:40 - 18:00	**Darlington**	**Ferri F.**		**Feingesicht**	