THE ASYMMETRIC PRODUCT,
A NEW CONSTRUCTION OF RADICAL F-BRACES
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Main Results

Let S and T be radical braces, let $b : T \times T \to S$ be a symmetric cocycle on $(T, +)$ with values in $(S, +)$, and suppose there exists an action of the group $(S, o)$ on the radical brace $T$ such that

$$b(t_1, t_2) s + b(t_1 + t_2, t_1) / 2 = b(t_1, t_2) s + b(t_1, t_2) / 2,$$

for all $s \in S$ and $t_1, t_2, t_3 \in T$. Then, the addition and the multiplication over $S \times T$ given by

$$b(s, t_1) + b(s, t_2) = b(s + b(t_1, t_2), t_1 + t_2)$$

$(s, t_1) = (s + b(t_1, t_2), t_1 + t_2)$

define a structure of radical brace on $S \times T$. We call this radical brace the Asymmetric Product of $T$ by $S$ (via $b$ and $o$) and denote it by $S \ltimes_T T$.

If $b$ is the null cocycle, then $S \ltimes_T T$ is the semidirect product of $T$ by $S$ (see (6) and also (3)).

Remark: The asymmetric product of two $F$-braces is not in general an $F$-brace.

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In the case of characteristic 2, the bilinearity of $b$ is not sufficient to obtain an $F$-brace. We have only a partial result that involves a bilinear map that is the polar form of a quadratic one.

Application

Let $F$ be of characteristic $p \neq 2$, and let $S$ and $T$ be radical $F$-braces. Let $b : T \times T \to S$ be a bilinear and symmetric map and suppose there exists an action of the group $(S, o)$ on the radical $F$-brace $T$ that satisfy the condition (1) of the previous theorem. Then the asymmetric product $S \ltimes_T T$ is a radical $F$-brace with the scalar multiplication given by

$$\lambda(s, t) = \left(\lambda s + \frac{\lambda(\lambda - 1)}{2} b(t_1, t_2), \lambda t\right)$$

for all $\lambda \in F, s \in S$ and $t \in T$.

References


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